1.2 Inductive Reasoning

Goal Use inductive reasoning to make conjectures.

VOCABULARY
Conjecture
Inductive reasoning
Counterexample

Example 1 *Make a Conjecture*

Complete the conjecture.

Conjecture: The sum of any two odd numbers is __?__.

Solution

Make sure you know the meaning of the conjecture.

A sum is ________________________.

The odd numbers are ____________.

List some examples and look for a pattern.

$1 + 1 =$ __ $\quad 5 + 1 =$ __ $\quad 3 + 7 =$ ___

$3 + 13 =$ ___ $\quad 21 + 9 =$ ___ $\quad 101 + 235 =$ ____

Each sum is _____. So you can make a conjecture.

Answer The sum of any two odd numbers is _____.

Follow-Up

Which of the following reasoning stages were used in Example 1? Look for a pattern   Make a conjecture   Verify a conjecture

Example 2 ***Make a Conjecture***

Complete the conjecture.

Conjecture: The sum of the first n odd positive integers is _?_.

Solution

The variable __ refers to the number of odd positive integers added together.

List some examples and look for a pattern.

For $n = 1$, the sum is $1 = 1 = \_\_^2$

For $n = 2$, the sum is $1 + 3 = \_\_ = \_\_^2$

For $n = 3$, the sum is $1 + 3 + 5 = \_\_ = \_\_^2$

For $n = 4$, the sum is $1 + 3 + 5 + 7 = \_\_\_ = \_\_^2$

For $n = 5$, the sum is $1 + 3 + 5 + 7 + 9 - \_\_\_ = \_\_^2$

For any n, the sum is ___.

Answer The sum of the first n odd positive integers is ___.

Follow-Up **Use the conjecture you made in Example 2 to find the sum of the first 8 odd positive integers.**

For $n = 8$, the sum is ___, or ___.

Check your answer by adding the first 8 odd positive integers.

__ + __ + __ + __ + __ + ___ + ___ + ___ = ___

Checkpoint **Complete the conjecture based on the pattern in the examples.**

1. The product of any two odd numbers is _____.

Examples: $1 \times 1 = 1$ $\quad 3 \times 5 = 15$ $\quad 3 \times 11 = 33$

$7 \times 9 = 63$ $\quad 11 \times 11 = 121$ $\quad 1 \times 15 = 15$

2. The product of the numbers $(n - 1)$ and $(n + 1)$ is _______.

Examples: $1 \cdot 3 = 2^2 - 1$ $\quad 3 \cdot 5 = 4^2 - 1$

$5 \cdot 7 = 6^2 - 1$ $\quad 7 \cdot 9 = 8^2 - 1$

$9 \cdot 11 = 10^2 - 1$ $\quad 11 \cdot 13 = 12^2 - 1$

Example 3 *Find a Counterexample*

Show the conjecture is false by finding a counterexample.

Conjecture: The sum of two numbers is always greater than the larger of the two numbers.

Solution

Here is a counterexample. Let the two numbers be 0 and 3. The sum is __, which is not greater than __, the larger of the two numbers.

Answer The conjecture is _____.

Follow-Up

Find a different counterexample for Example 3.

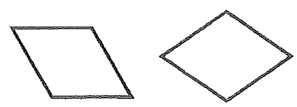
Example 4 ***Find a Counterexample***

Show the conjecture is false by finding a counterexample.

Conjecture: All shapes with four sides of the same length are squares.

Solution

Two counterexamples are shown at the right. These shapes have four sides of the same length, but they are not ________.



Answer The conjecture is ______.

✓ *Checkpoint* Show the conjecture is false by finding a counterexample.

3. If the product of two numbers is even, then the numbers must be even.

4. If a four-sided shape has two sides the same length, then it must be a rectangle.